1 1 1 1
-1 -1 -1 -1

$4 + -4 = 0$

1 1 1 1
-1 -1

$0 - -2 = 2$

INTEGERS

Temperature Stages in the Production of Ice-cream

Pasteurisation

80°C

0°C

Aging

-40°C

Ready for sale

Hardening

+ 9

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6
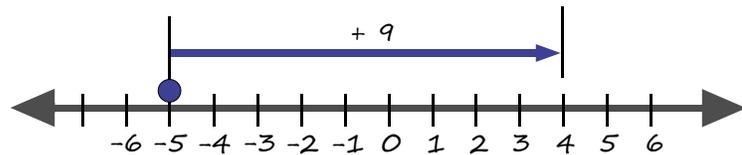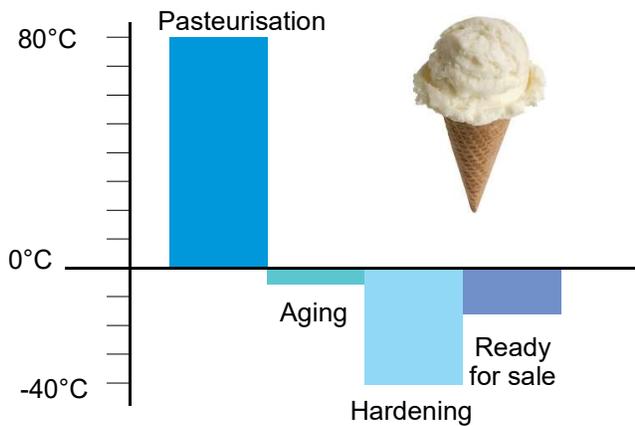
Integers: *The set of whole numbers and their opposites*
{…-3, -2, -1, 0, 1, 2, 3, . .}

Kim Freeman

Comparing and Ordering Integers

Negative integers   Positive integers

–8 –7 –6 –5 –4 –3 –2 –1 0 1 2 3 4 5 6 7 8

Less than   Greater than

Zero is neither negative nor positive
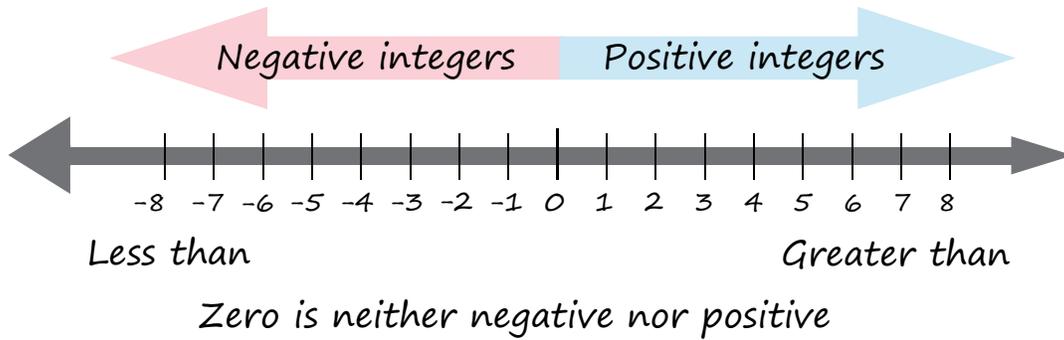
Examples

Water freezes at 0°C but some insects can still survive by expelling excess water and producing a chemical that lowers the temperature at which the water in their body freezes. This lowered temperature is called "supercooling".

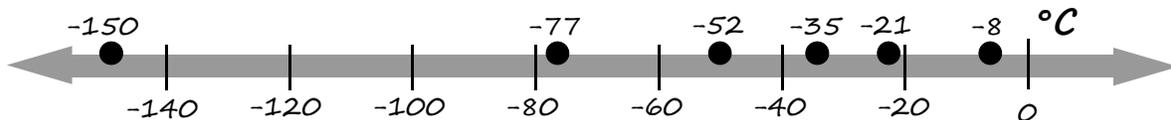| Insect	Supercooling point
Arctic Beetle	–52°C
Gall Beetle	–35°C
Goldenrod Gallfly	–8°C
Red Flat Bark Beetle	–150°C
Snow Flea	–21°C
Woolly Bear Caterpillar	–70°C
Upis Beetle	–77°C

Snow Fleas

Woolly bear caterpillar

Red flat bark beetle

The number line shows these temperatures ordered from least to greatest.

–150   –77   –52 –35 –21   –8   °C

–140   –120   –100   –80   –60   –40   –20   0

Less Than (<) or Greater Than (>)

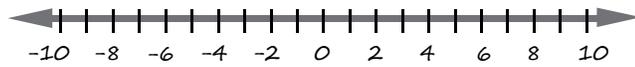–7 > –12   –5 < 0

–12   –10   –8   –6   –4   –2   0

–2 < 2

–6   –4   –2   0   2   4   6

3 > –5
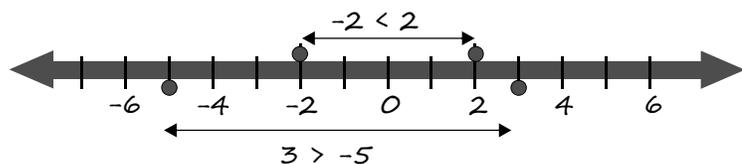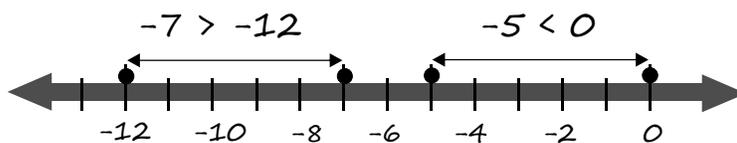
Exercises

For each of the following, graph the integers from least to greatest.

1. –9, 4, –3, 1, 0, 8

2. –11, 5, –7, 0, –2

3. 15, –7, –4, 8, –5

4. 30, 50, –10, –5, 40

5. –25, –30, –15, 10

The table below shows the daily high temperatures at Antartica's Scott Base during one week in March.

Day	Temperature
Sun	-16°C
Mon	-17°C
Tue	-9°C
Wed	-13°C
Thu	-18°C
Fri	-25°C
Sat	-21°C

6. Did the temperature increase or decrease Sunday to Monday?

7. Did the temperature increase or decrease from Friday to Saturday?

8. Which was the coldest day?

9. Which was the hottest day?

10. What was the weekly mean temperature?

Copy and complete the statements below using a > or < sign.

11. –7   5

12.   –9 __ –15

13. 0 __ 5

14. 5   –15

15.   –20 __ 15

3

16. In 1989, data collected by the Voyager spacecraft showed the surface temperature of Triton, Neptune's largest moon, to be –236°C. Recent data from the Hubble telescope showed the temperature to be –234°C. Did the Hubble data indicate a temperature less than or greater than the one based on the Voyager data?
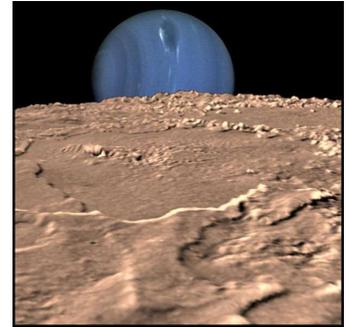
Neptune shown on Triton's horizon.

17. A scuba diver studying marine life is 2 metres below sea level. From that depth, the diver descends another 15 metres to the ocean floor. After 15 minutes studying the area he rises 8 metres and rests to avoid decompression illness. Where is the diver relative to sea level?

18. Your bank account statement shows an overdraft of $25. You deposit $100, then spend two amounts at the mall: one for $12 and one for $55. What is your new balance?

19. The lowest temperature ever recorded on Earth was measured at –89°C, and was recorded at Lake Vostok in Antartica. The lowest temperature ever recorded in New Zealand was in Ophir, Central Otago and was about 62°C higher. What was the record low temperature in New Zealand?

4

20. Kick-'em-Jenny is an underwater volcano in the Caribbean Sea. Each year, eruptions cause the volcano to grow. In 1962, the summit elevation of Kick-'em-Jenny was measured at -235 meters. In 2008, the summit elevation was measured at -171 meters.
By how many meters did the elevation of the volcano change?

The 1939 eruption of Kick `Em Jenny probably looked similar to this eruption of the Kavachi Submarine Volcano in the Solomon Islands. (Photo by Pamela Brodi, 2000)

21. There are four stages in the production of ice cream. First, the mix is raised to a temperature of 80°C to destroy any bacteria. This is called pasteurisation. Next, the temperature is lowered to -5°C for the mix to age. Flavours are then added and the temperature is lowered to -40°C to harden the ice cream. Finally, the ice cream is moved to a freezer with a temperature of -15°C.
Use this information to calculate the change in temperature between the four consecutive stages.

Temperature Stages in the Production of Ice-cream

5
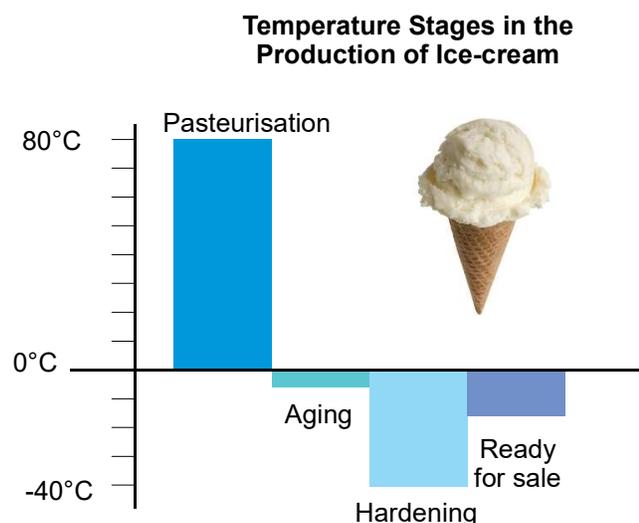
22. Ethylene glycol is a chemical that, added to water, lowers its freezing point. Solution 1, is one part ethylene glycol and three parts water. The freezing point is −11°C. The freezing point of solution 2, which is two parts ethylene glycol and two parts water, is −35°C.
Which solution has the lower freezing point? How much lower is it?
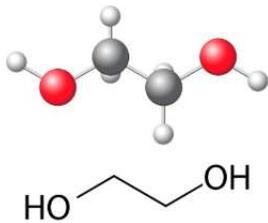
23. Surtsey is a volcanic island off the coast of Iceland. It was initially formed by a volcanic eruption in 1963 at 130m below sea level. By 1967 it had reached a maximum elevation of 174m. Since that time it has been reducing in size due to wind and wave erosion.
What was the change in elevation from 1963 to 1967.

Surtsey Island

24. Food scientists have tested the effects of freezing cheese and tomato puree filling in lasagne. The filling was frozen to a temperature of −21°C and then raised by 235°C.
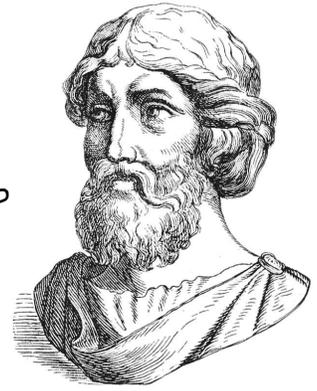What was the final temperature of the filling?

Lasagne - better eaten hot than cold.

25. Pythagoras was born about 582 BC.

Sir Isaac Newton was born in 1643 AD.

How many years apart were they born.?

26. The highest point of Mt. Everest is at 29 028 feet.
The lowest point of the Dead Sea is at 1 312 feet below sea level.
What is the difference in the height and depth between Mt. Everest
and the Dead Sea?

27. The Roman Civilization began in 509 B.C. It ended in 476 A.D.
How long did Roman Civilization last?

28. At room temperature, the metal mercury is a liquid.
The melting point of mercury is –39°C.
By contrast, the freezing point of alcohol is–114°C.
How much warmer is the melting point of mercury than the freezing
point of alcohol?

Hydrogen

Oxygen

Carbon

Alcohol

80
Hg
Mercury
200.59
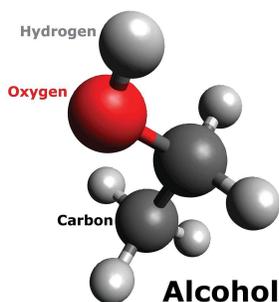
Adding and Subtracting Integers

You can use a number line to add or subtract integers.
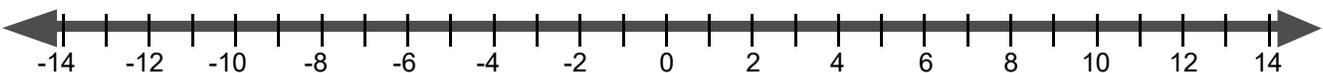For example:

−5 + 9 = 4

8 − 15 = −7
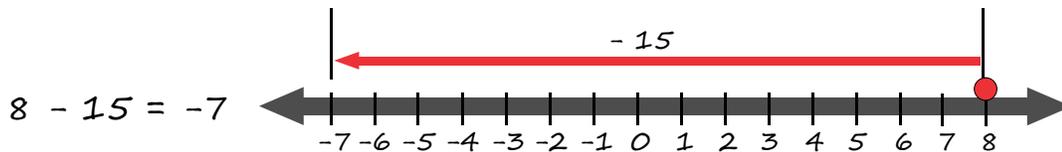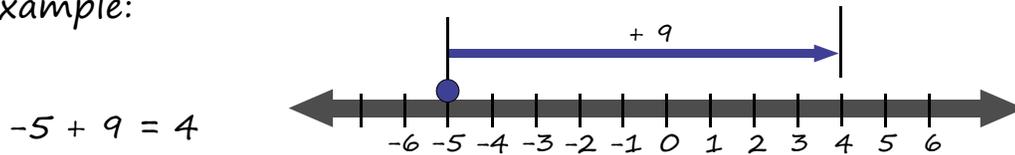
Exercises

Add the following integers.

1. −7 + 8 =

2. −5 + 3 =

3. −2 + 9 =

4. −10 + 7 =

5. −13 + 19 =

6. −8 + 17 =

7. −3 + 3 =

8. −14 + 27 =

9. −12 + 4 =

10. −9 + 19 =

11. −6 + 17 =

12. −11 + 6 =

13. 8 + 7 =

14. 0 + 12 =

15. −1 + 15 =

Subtract the following integers.

16. 12 − 18 =

17. 9 − 9 =

18. −2 − 8 =

19. −6 − 9 =

20. 14 − 27 =

21. 8 − 15 =

22. 0 − 5 =

23. −1 − 10 =

24. 6 − 8 =

25. 3 − 11 =

26. 13 − 7 =

27. 9 − 12 =

28. −2 − 4 =

29. −10 − 4 =

30. 10 − 21 =

Write these sentences as sums and calculate the answers.

31. The temperature was 12°C during the day but fell 19°C that night.

32. The car was parked 5 floors underground. Jones traveled in the lift from this floor to the 8th floor of the building.

33. Mr Greene had $5000 in the bank. He spent $5400 on a car. How much does he now have in his bank account?

34. One of the most extreme temperature changes in history occurred in Alberta Canada when in one hour, the temperature rose from -19°C up to 22°C. What was the change in temperature?

35. The hottest temperature recorded in the world was in Libya in 1922, when air temperature in the Sahara desert reached 58°C. The ground temperature was measured at 66°C. What was the difference between the air temperature and the ground temperature?

36. In 1960, the US Navy sent the submersible mini-submarine Trieste down into the depths of the Marianas Trench (the deepest part of the ocean). It touched the bottom at 10,923m. Mount Everest (the tallest mountain) is measured at 8850 m. If Mount Everest was moved to the bottom of the Marianas Trench how far below sea level would its peak be?
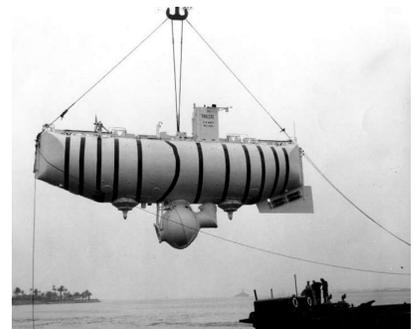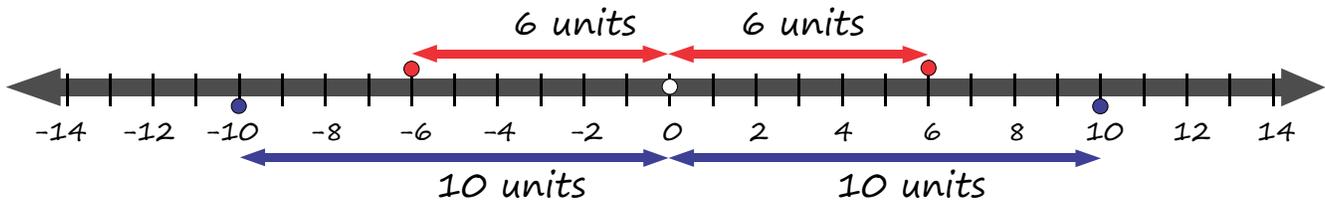

The USS Trieste (1958)

Integer Opposites

Two numbers are opposites if they are the same distance from 0 on the number line. When you add opposites the sum is 0.

For example: The opposite of 6 is –6

$$6 + (-6) = 0$$

6 units   6 units

-14   -12   -10   -8   -6   -4   -2   0   2   4   6   8   10   12   14

10 units   10 units

The opposite of –10 is 10

$$-10 + 10 = 0$$

Examples

Which number value does each diagram represent?

1 1 1 1 1 1
-1 -1 -1 -1 -1 -1 -1 -1 -1   = –3

1 1 1 1 1
-1 -1 -1 -1   = 1

Integer opposites sum to equal zero. Therefore If you have the same number of 1s as –1s then you have zero.

1 1 1 1 1
-1 -1 -1 -1 -1   = 0

Exercises

Add the following:

1. (−6) + 6 =

2. 10 + (−10) =

3. (−8) + 8 =

4. 2 + (−2) =

5. 6 + (−6) =

6. (−10) + 10 =

Add the following:

7. 8 + (−8) + 12 =

8. (−2) + 2 − 2 =

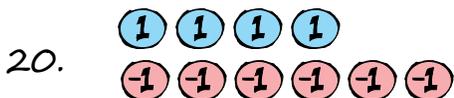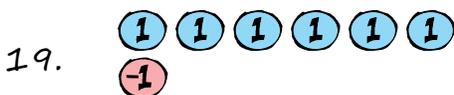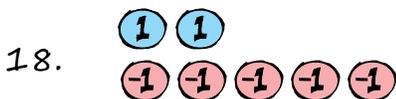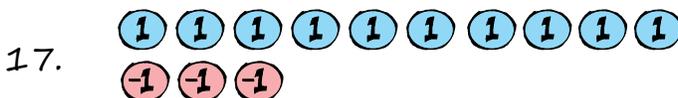9. −7 + 10 + 7 =

10. (−5) + 8 + 5 =

11. 3 + (−3) + 9 =

12. 15 + 7 + (−7) =

Which number do these diagrams represent?

13.

14.

15.

16.

17.

18.

19.

20.

11

$1\ 1\ 1\ 1\ 1\ 1\ 1$
$-1\ -1\ -1\ -1\ -1\ -1\ -1\ -1\ -1$ $\quad 7 + (-9) = -2$

$1\ 1\ 1$
$-1\ -1\ -1\ -1\ -1\ -1\ -1\ -1$ $\quad 3 + (-8) = -5$

$-1\ -1$
$1\ 1\ 1\ 1\ 1\ 1\ 1\ 1$ $\quad -2 + 8 = 6$
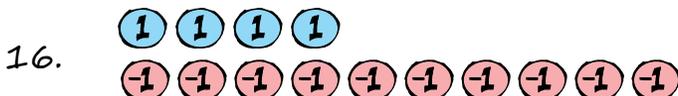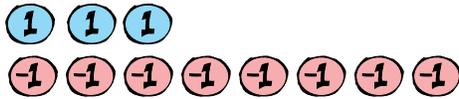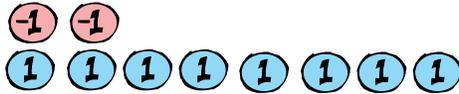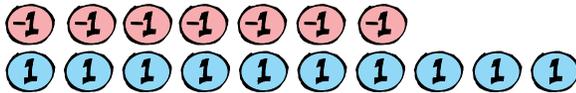
$-1\ -1\ -1\ -1\ -1\ -1\ -1$
$1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1$ $\quad -7 + 10 = 3$

Integer opposites sum to equal zero.

If you have the same number of 1s as −1s then you have zero.

$1\ 1\ 1$
$-1\ -1\ -1$ $= 0$ $\qquad$ 1
-1 $= 0$ $\qquad$ $1\ 1\ 1\ 1\ 1\ 1$
$-1\ -1\ -1\ -1\ -1\ -1$ $= 0$

$1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1$
$-1\ -1\ -1\ \cancel{-1}\ \cancel{-1}$ $\quad 5 - (-2) = 7$

$1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1$
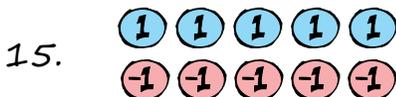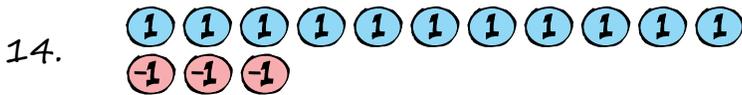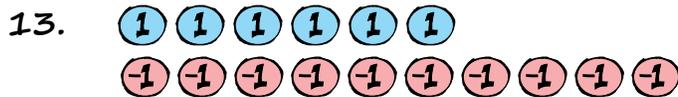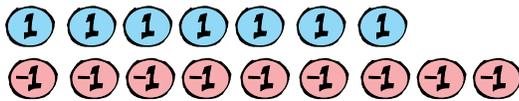$\cancel{-1}\ \cancel{-1}\ \cancel{-1}\ \cancel{-1}\ \cancel{-1}\ \cancel{-1}$ $\quad 4 - (-6) = 10$

$-1\ \cancel{-1}\ \cancel{-1}$ $\quad -3 - (-2) = -1$

$1\ 1$
$\cancel{-1}\ \cancel{-1}\ \cancel{-1}\ \cancel{-1}\ \cancel{-1}\ \cancel{-1}\ \cancel{-1}\ \cancel{-1}\ \cancel{-1}\ \cancel{-1}$ $\quad -8 - (-10) = 2$
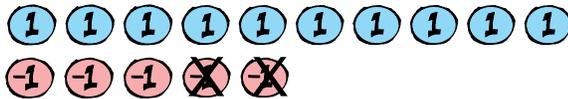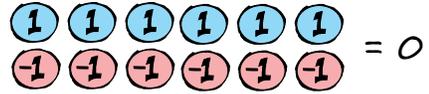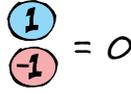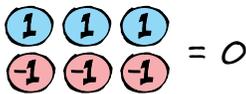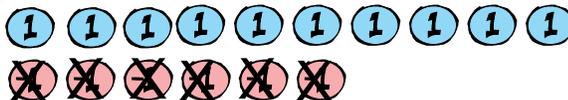
$-9 - (-13) = 4$
$1\ 1\ 1\ 1$
$\cancel{-1}\ \cancel{-1}\ \cancel{-1}\ \cancel{-1}\ \cancel{-1}\ \cancel{-1}\ \cancel{-1}\ \cancel{-1}\ \cancel{-1}\ \cancel{-1}\ \cancel{-1}\ \cancel{-1}\ \cancel{-1}$

Subtracting a negative integer is the same as adding the integer opposite.

$-9 - (-13) = -9 + 13$
$\qquad\qquad\ = 4$

Adding and Subtracting Negative Integers

Exercises

Use the diagrams to help add the following integers.

1. (diagram) $3 + (-10) =$

2. (diagram) $10 + (-6) =$

3. (diagram) $9 + (-7) =$

4. (diagram) $4 + (-10) =$

5. (diagram) $10 + (-5) =$

6. (diagram) $5 + (-4) =$

7. (diagram) $-8 + 0 =$

8. (diagram) $-2 + 10 =$

9. (diagram) $-7 + 10 =$

10. (diagram) $-9 + 5 =$

11. (diagram) $-7 + 2 =$

12. (diagram) $-10 + 1 =$

13
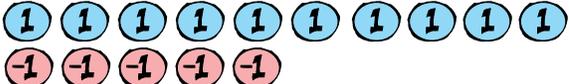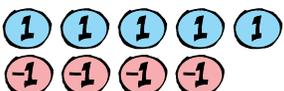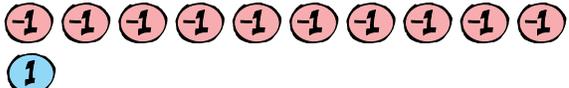
Use the diagrams to help subtract the following integers.
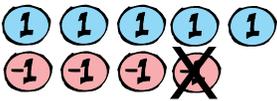
13. $1 - (-1) =$
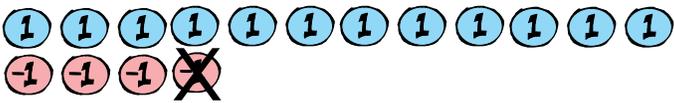
14. $8 - (-1) =$

15. $4 - (-4) =$

16. $3 - (-2) =$
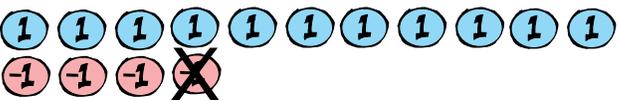
17. $7 - (-1) =$

18. $2 - (-8) =$
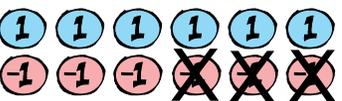
19. $0 - (-3) =$
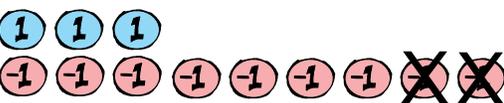
20. $-6 - (-2) =$

21. $-9 - (-9) =$

22. $-5 - (-8) =$

23. $-10 - (-7) =$

24. $-7 - (-3)$

14
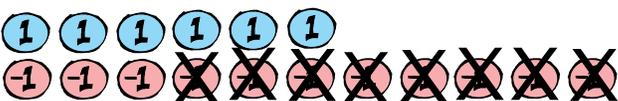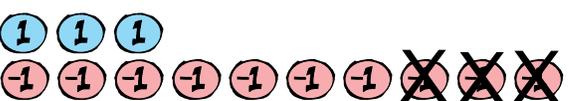
Exercises

Calculate

1. −3 + (−5) =

2. −9 + (−2) =

3. 8 + (−2) =

4. −5 + (−8) =

5. −3 + 5 =

6. −9 + 4 =

7. −4 + 9 =

8. 10 + (−8) =

9. 6 + (−13) =

10. 13 + (−9) =

11. −5 + (−7) =

12. −13 + 5 =

13. 13 + (−4) =

14. −8 + (−9) =

15. −5 + (−6) =

16. −14 + 8 =

17. −1 + 10 =

18. 9 + (−3) =

19. 10 + (−20) =

20. −2 + 15 =

21. −12 + 30 =

22. −5 + −6   =

23. 0 + (−8) =

24. −12 + (−3) =

25. −4 − 10   =

26. 5 − 9 =

27. −1 − 6 =

28. −8 − 8 =

29. −6 − (−5)

30. 12 − (−3)

31. 5 − (−8)

32. −8 − (−3)

33. −6 − (−7)

34. 9 − (−5)

35. −6 − (−4)

36. −7 − (−7)

37. −11 − 6

38. −15 − (−8)

39. −3 − 15

40. 0 − (−9)

41. −14 − (−4)

42. −3 − (−9)

43. −9 − (−6)

44. 1 − (−10)

If m = −5, evaluate:

45. 15 − m

46. m − 12

47. −7 − m

48. 20 + m

49. 8 + (−m)

50. −9 + (m)

More Integer Arithmetic

$$- 6 + 13 = 7$$

$$- 1 + (- 6) = -7$$

$$-5 - (-9) = -5 + 9$$
$$= 4$$
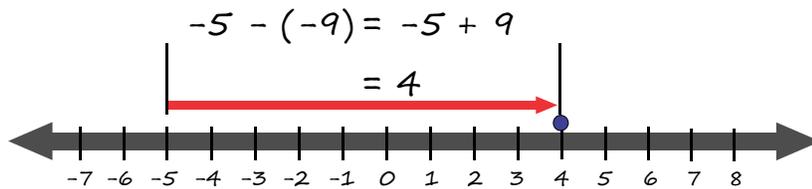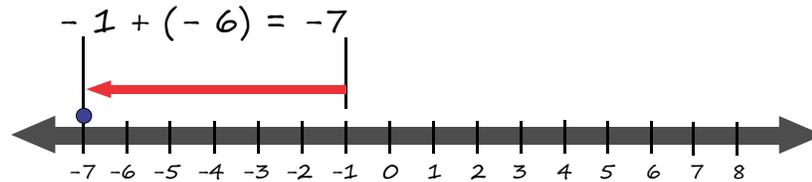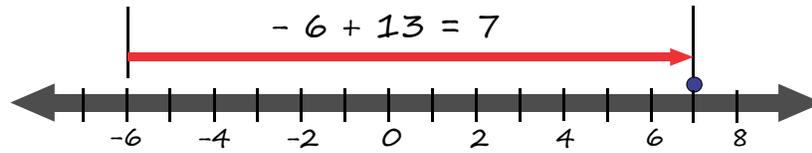
A number line can still be used for integer arithmetic.
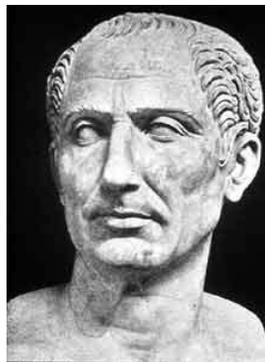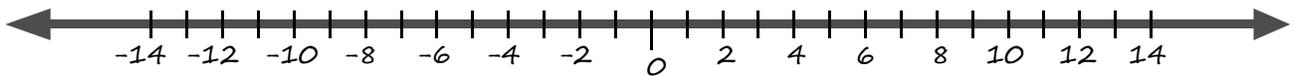
Two statue depictions, one of Julius Caesar and the other a young Cleopatra.

Exercises

1. Julius Caesar was born in 100 BC and died in 44 BC.
    His mistress Cleopatra was born in 69 BC and died in 30 BC.
    a. How old was Caesar when he died?

    b. How old was Cleopatra when she died?

    c. What was the age difference between Caesar and Cleopatra?

2. A cliff top overlooking the ocean is 123 m above sea level. The sea floor at the foot of the cliff is 15 m below sea level. A stone is dropped from the top of the cliff and falls to the sea floor. Write an expression that best represents the distance the rock has fallen.

3. At takeoff (T) minus 31 seconds the onboard computers of the space shuttle Atlantis take over the launch sequence from the ground network. At T minus 7 seconds, the shuttle's main engines ignite. At T minus 0 seconds, the solid rocket boosters ignite and we have liftoff for Atlantis. At T plus 156 seconds the solid rocket boosters are exhausted of fuel and they detach from the orbiter and fuel tank. At T plus 9 min, the external tank separates from the orbiter and at T plus 10 min 30 seconds the orbital maneuvering system engines fire to place the Atlantis in a low orbit. Finally, at T plus 45 min, the orbital maneuvering system engines fire again to place the shuttle in a higher, circular orbit (about 250 miles/400 km). The space shuttle is now in outer space.

What is the time difference between when the onboard computers take over the launch sequence and when the solid rocket boosters detach from Atlantis?

Liftoff for Atlantis

4. According to the Guinness Book of World Records, Verkhoyansk, the river port in North Eastern Siberia, has the most extreme climate on the planet. The average winter temperature is –50°C while the average summer temperature is 14°C. What is the difference between the coldest and hottest average temperatures.

The Verkhoyansky range.

5. The tallest mountain in the world (when measured from base to peak) is Mauna Kea (white mountain) in Hawaii. Its base is 6000 metres below sea level and it rises 10207 metres.
What is the height of the peak above sea level?

Mauna Kea (The White Mountain) named because of seasonal snow.

6. Business profits are expressed as a positive number and are usually referred to as operating in the black. A business loss is a negative number and is usually referred to as operating in the red.
Look at the 6 month profit – loss graph below and find the sum of the profits and losses.

Monthly Profits and Loss Graph

$10 500 $5 400   Mar   Apr $4 200 $2 500

Jan   Feb   Mar   Apr   May   Jun

-$6 100

-$12 800

18
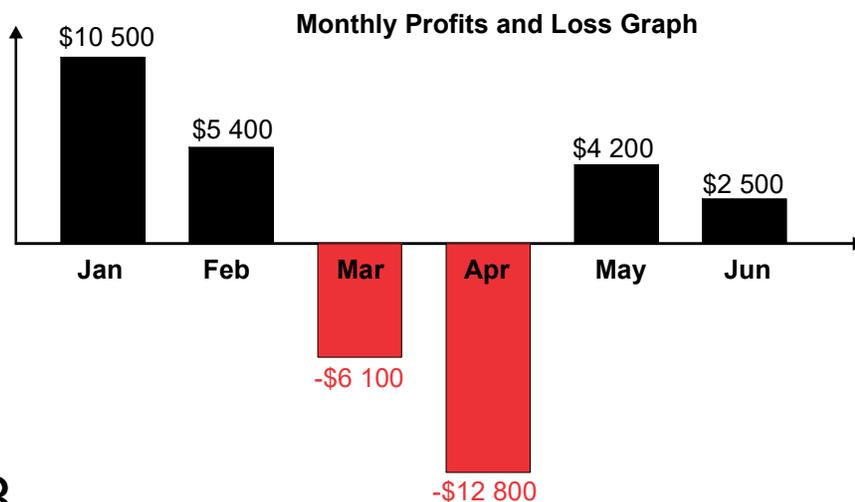
7. A company's accounts sheet is shown in the table on the right. It shows the profit and loss results for an 8 month period.
What was the overall profit or loss?

Month	Profit	Loss
Aug	$30 000	
Sept		-$50 000
Oct		-$10 000
Nov	$20 000	
Dec	$15 000	
Jan		-$20 000
Feb	$10 000	
Mar	$15 000	

8. A country's exports and imports are usually collated each month by the government statistics department. If more money is received from exports than spent on imports then the country is running a trade surplus. If more money is spent on imports than received from exports then the country is running a trade deficit. A graph showing 7 months of trade surpluses and deficits for New Zealand could look similar to that below:

Graph of Monthly Trade Surplus - Deficit
Figures indicate millions ($)

Jan   Feb   Mar   Apr   May   Jun   Jul
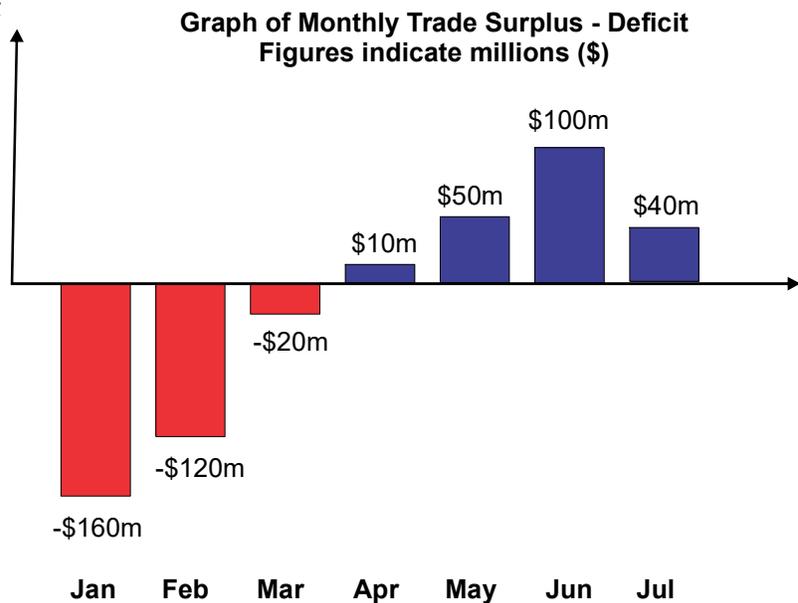
a. What is the change in trade figures between Feb and March?

b. What is the change in trade figures between March and May?

c. Calculate the total surplus or deficit for the 7 month period.

9. Lake Taupo in the middle of the North Island, New Zealand, is constantly having its level monitored. The level rises and falls due to rainfall, winter snow melt and extended dry periods. Over a 4 month period the levels were recorded as: April rise of 2 metres, May rise of 1 metre, June fall of 5 metres, July rise of 3 metres. How much has the level changed over the 4 month period?

Lake Taupo with the volcanoes of Tongariro National Park beyond

10. A student who has $180 in her account receives a $750 tax refund. She then pays university course fees of $550. How much does she now have in her account?

11. Claudia has a –$467 balance on her credit card. She returns a sweater worth $129 to the store. How much does she now owe on her credit card?

12. At the beginning of August, Sabine's credit card account shows a debt of $470. She makes a payment of $45 but then makes further purchases of $160. At the end of August she makes a further payment of $500. What is the balance of her account at the beginning of September?

PayPal allows payments and money transfers to be made through the Internet.

13. The surface temperature on Mars has been measured as –128°C on a polar night and 27°C at midday when it has been at its closest orbit point to the sun. Find the range of the temperatures i.e. the difference between the high and low temperatures.

14. When playing golf the following terms apply:
   Par, the number of shots that it should take to hole the golf ball.
   Birdie, 1 shot below par
   Eagle, 2 shots below par
   Bogie, 1 shot above par
   Double Bogie, 2 shots above par

   The Ngunguru Golf Course is a nine hole golf course. Ngunguru is "a place in the sun", 20 minutes from Whangarei. Par for the course is 29 shots. In one particular game a player scored the following: One hole on par, two birdies, one eagle, four bogies and one double bogie. How many points above or below par was the player and what was their final score?

Calculate:

15. 2 + (–7) =	23. –7 – 0 =	31. –15 – (–45) =
16. –5 + 2 =	24. –13 – 2 =	32. –23 – (–16) =
17. –11 + 5 =	25. 0 + (–27) =	33. –33 – 44 =
18. 4 + (–2) =	26. –17 + (–25) =	34. –20 – (–20)
19. –6 + 7 =	27. –24 + (–9) =	35. –26 + 15
20. 8 – (–8) =	28. 19 + (–15) =	36. –31 + (–12)
21. 3 – (–5) =	29. –27 + 18 =	37. 13 + (–9)
22. –4 – (–6) =	30. 53 – (–18) =	38. 27 + (–4)

39. The stock market in New Zealand is run by the NZSX. Company stocks are bought and sold and their price fluctuates each day. At the end of each day, week and month the average price across all stocks is reported and the average price will be either up or down from the previous period. Over a 6 week period a class took note of the NZSX trading. The results are listed below:

Week 1: Down 13 points
Week 2: Down 16 points
Week 3: Up 36 points
Week 4: Down 11 points
Week 5: Up 19 points
Week 6: Up 20 points

Outside the New Zealand
Stock Exchange.

By how many points has the stock market fallen or risen over the whole 6 week period?

40. Add up all the integers from –50 to 50. What is the final total?

41. 18 + (–21) =

42. 23 – 38 =

43. –21 + 25 =

44. –50 – (–9) =

45. 23 – (–23) =

46. –32 – 10 =

47. 25 + (–31) =

48. –19 – 8 =

49. 29 – (–16) =

50. –15 – 21 =

Multiplying and Dividing Integers

Example

An investor owns shares of stock in an electronics company, an energy company and a construction company. Over one year the values of the shares change as shown in the table below.
Calculate the total change in value:

Stock Portfolio

Stock	Number of shares	Change in value of one share	Total value change
Electronics	500	decreased $2	$1 000 loss
Energy	300	increase $4	$1 200 gain
Construction	200	decrease $3	$ 600 loss

Total Change in value = (−$400)

i.e. $ 400 loss

The bronze statue of the Merrill Lynch Bull in New York's financial district is usually identified as symbol of Wall Street, home of the New York Stock Market.

Exercises

1. During a chemical reaction the temperature in a test tube decreased by 2°C every minute until 9:23am. If the temperature was 15°C at 8:55am what was the temperature at 9:23am?

2. Scientists were asked to test a Marine GPS device that was going to be used in Antarctica. They placed the device in a test chamber set at a temperature of 20°C. Each minute they lowered the temperature in the chamber by 3°C.

   a. Write an integer that represents the change in the temperature in the test chamber in 1 minute.

   b. Write a product that represents the total change in temperature after 7 minutes.

   c. Calculate the temperature in the chamber after 10 minutes.
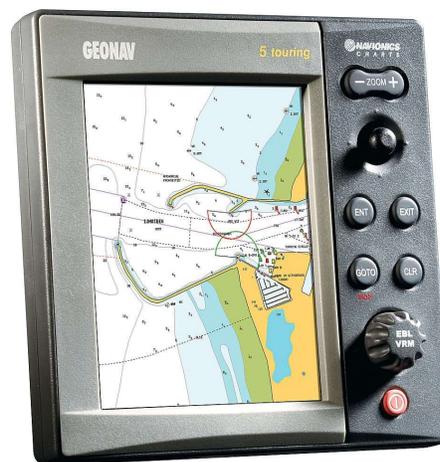
A typical marine GPS device.

3. The colour of fireworks are determined by the heat being generated. At 480°C the colour is faint red, at 580°C the colour is dark red, at 730°C bright red, at 930°C bright orange, at 1100°C pale yellow, 1300°C yellowish white and 1400°C white. If a set of fireworks is ignited and their temperature increases by 280°C every second, how long will it take before the fireworks are the colour white?

Fireworks across the Sydney Harbour Bridge. Fireworks are the result of a chemical reaction

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4. As altitude increases, the air pressure decreases. This means that the boiling point of water also decreases. At sea level, water boils at 100°C. As a rough guide, the boiling temperature decreases by 1°C as altitude increases by 300 metres.

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a. Mt Taranaki is found in New Zealand's North Island. It has a peak of 2518 metres. If climbers stopped at 2100 metres and boiled some water for a coffee break what would be the temperature of the boiling water?

Mt Taranaki, New Zealand

. b. Mt Cook is New Zealand's highest mountain with a peak at 3755 metres above sea level. If climbers were boiling water at a height of 3600 metres what would the boiling temperature of the water?

5. McMurdo Station, in Antarctica, is located on the southern tip of Ross Island on the shore of McMurdo Sound in New Zealand territory, 3,500 km due south of New Zealand. The station is the largest community in Antarctica and can support up to 1,258 residents. The table below shows average temperatures at McMurdo Station from April to September. Find the mean of the temperatures.

Month	Apr	May	Jun	Jul	Aug	Sep
Av Temp °C	–19°	–21°	–19°	–22°	–24°	–21°

Smoothing snow at McMurdo Sound to prepare for aircraft landings.

Multiplying Two Integers

SAME Signs - product is POSITIVE

e.g. $\quad -4 \times -14 = 56$

$\quad\quad\quad 5 \times 15 = 60$

DIFFERENT signs - product is NEGATIVE

e.g. $\quad 7 \times -8 = 48$

$\quad\quad -20 \times 4 = -80$

The product of any integer and 0 is 0

e.g. $\quad -13 \times 0 = 0$

$\quad\quad\quad 0 \times 28 = 0$

Dividing Two Integers

SAME signs - **Quotient** is POSITIVE

e.g. $\quad 16 \div 8 = 2$

$\quad\quad\quad -27 \div -9 = 3$

DIFFERENT signs - quotient is NEGATIVE

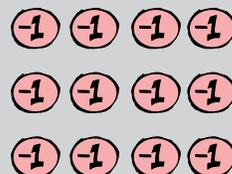*e.g.* $\quad -24 \div 6 = -4$

$\quad\quad\quad 30 \div -10 = 3$
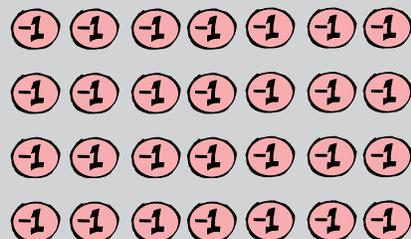
$4 \times (-7) = -28$

$-28 \div 4 = -7$

each row has a value of -7

$-28 \div -7 = 4$

there are 4 rows of -7

$3 \times (-4) = -12$

$-12 \div 3 = -4$

$-12 \div -4 = 3$

Calculate:

6.	$8 \times -11 =$	16.	$-24 \div 8 =$	26.	$-15 \times -4 =$
7.	$-6 \times -8 =$	17.	$0 \div -20 =$	27.	$24 \div -6 =$
8.	$-5 \times -7 =$	18.	$-36 \div -6 =$	28.	$-18 \times 3 =$
9.	$0 \times -20 =$	19.	$28 \div -7 =$	29.	$-54 \div -2 =$
10.	$20 \times -6 =$	20.	$40 \div -4 =$	30.	$20 \times -7 =$
11.	$-9 \times -4 =$	21.	$-39 \div -13 =$	31.	$-200 \div -5 =$
12.	$-4 \times 7 =$	22.	$96 \div -3 =$	32.	$-90 \times -9 =$
13.	$-15 \times 3 =$	23.	$-42 \div -7 =$	33.	$-76 \div 4 =$
14.	$4 \times -18 =$	24.	$-98 \div 2 =$	34.	$-6 \times -5 \div -3 =$
15.	$25 \times -5 =$	25.	$-64 \div 16 =$	35.	$-8 \div 2 \times -3 =$

36. To find the difference between average temperature, and actual temperature you can use the expression: **D = H – A**

where   H is the historical mean

A is the actual measured mean temperature.

Use the table below to find the difference between the average historical temperatures and the measured temperature means for Auckland during one February. Then find the mean off all the differences.

Temperatures for Auckland during February

Week	1	2	3	4
Historical Mean	26°	28°	26°	28°
Measured Mean (°C)	24°	25°	27°	24°

Sailing into Auckland City.

37. A MIR submersible is a small type of submarine. It can dive at a rate of 25 metres each minute. On one particular day a MIR takes 240 minutes to reach the lowest point to which it can safely dive.

a.   What is its depth at that point?

b.   How long would the MIR take to dive to –425 metres (i.e. 425 metres below sea level) if it was diving 25 metres each minute?

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A Russian deep-manned Submersible MIR. It provides valuable information for oceanographic researchers.

Calculate:

38. $(-3 \times 5) \times 4 =$

39. $(8 \times -6) \times -2 =$

40. $(-6 \times -5) \times -2 =$

41. $-2 \times (-8 \times -2) =$

42. $(-7 \times -2) \times -3 =$

43. $-3 \times (9 \times 10) =$

44. $(4 - 3) \times (5 - -1) =$

45. $(-5 - 8) \times (6 - 10) =$

If $x = -5$, find the value of:

46. $-10x^2 =$

47. $\dfrac{75}{x^2} =$

48. $-(8x^2) =$

49. $\dfrac{4x^2}{-10} =$

50. $x^2 + 5x^2 =$

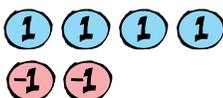# Subtract a Negative = Add a Positive

Sometimes it is hard to visualise why when subtracting a negative number, you add a positive. This unit of work illustrated it using integer balls.

This example = 0   ① ① ① ①
                  -1 -1 -1 -1

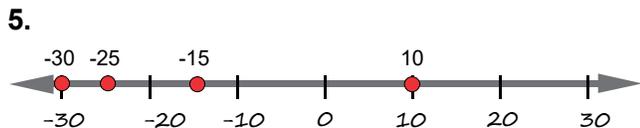There are the same number of negative and positive balls.

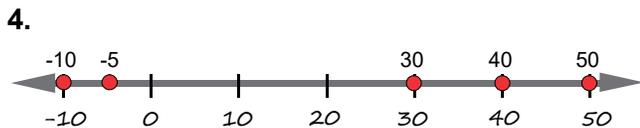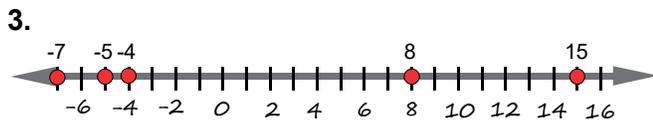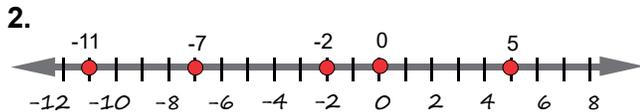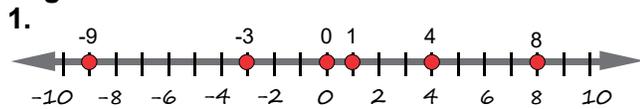Subtracting -2 equals 2   ① ① ① ①
                          -1 -1

This means $0 - (-2) = 0 + 2$
$$= 2$$

Another way of thinking about this is using a banking example. If you owe the bank $100 then you have -$100 in your account. If your parents subtract your debt (i.e. they pay it off for you) then the account is back to $0.

     i.e. $-\$100 - (-\$100)$

          $= -\$100 + \$100$

          $= \$0$

The Answers

1.

2.

3.

4.

5.

6. Temperature decreased by 1°C
7. Temperature increased by 4°C
8. Coldest day was Friday (-25°C)
9. Hottest day was Tuesday (-9°C)
10. -(16+17+9+13+18+25+21) ÷ 7
= -119°C ÷ 7
= -17°C
11. -7 < 5
12. -9 > -15
13. 0 < 5
14. 5 > -15
15. -20 < 15
16. Voyager = -236°C, Hubble = -234°C
The Hubble is 2°C greater.
17. The diver moves from -2m to -17m
Then rises from -17m up 8m = -9m
The diver is 9m below sea level.
18. -$25 + $100 = $75
$75 - $12 - $55 = $8 left in account.
19. The calculation is -89 + 62 = -27
Ophir must have been -27°C
20. The calculation is -235 + 64 = -171
The elevation changed by +64m
21. Stage 1 to Stage 2 = 80°C to -5°C
This is a change of -85°C
Stage 2 to Stage 3 = -5°C to -40°C
This is a change of -35°C
Stage 3 to Stage 4 = -40°C to -15°C
This is a change of +25°C

22. Sol 2 has lower freezing point by -24°C
-11°C - 24°C = -35°C
23. 1963 elevation = -130 m
1967 elevation = +174 m
-130 m + 304 m = 174 m
Change in elevation is +304 m
24. -21 + 235 = 214
Final temperature = 214°C
25. 582 + 1643 = 2225 years
26. 1312 + 29028 = 30340 feet
27. 509 + 476 = 985 years
28. -114 - (-39) = 75°

1. -7 + 8 = 1
2. -5 + 3 = -2
3. -2 + 9 = 7
4. -10 + 7 = -3
5. -13 + 19 = 6
6. -8 + 17 = 9
7. -3 + 3 = 0
8. -14 + 27 = 13
9. -12 + 4 = -8
10. -9 + 19 = 10
11. -6 + 17 = 11
12. -11 + 6 = -5
13. 8 + 7 = 15
14. 0 + 12 = 12
15. -1 + 15 = 14
16. 12 - 18 = -6
17. 9 - 9 = 0
18. -2 - 8 = -10
19. -6 - 9 = -15
20. 14 - 27 = -13
21. 8 - 15 = -7
22. 0 - 5 = -5
23. -1 - 10 = -11
24. 6 - 8 = -2
25. 3 - 11 = -8
26. 13 - 7 = 6
27. 9 - 12 = -3
28. -2 - 4 = -6
29. -10 - 4 = -14
30. 10 - 21 = -11
31. 12°C - 19°C = -7°C
32. -5 + 13 = 8
33. $5000 - $5400 = -$400 ($400 overdraft)
34. -19 + 41 = 22
35. 58 - 66 = -8, i.e. a -8°C difference
36. -10923 + 8850 = -2073 m
2073 metres below sea level.

Page 11
1. (-6) + 6 = 0
2. 10 + (-10) = 0
3. (-8) + 8 = 0
4. 2 + (-2) = 0
5. 6 + (-6) = 0
6. (-10) + 10 = 0
7. 8 + (-8) = 0
8. (-2) + 2 = 0
9. -7 + 10 + 7 = 10
10. -5 + 8 + 5 = 8
11. 3 + (-3) + 9 = 9
12. 15 + 7 + (-7) = 15
13. 6 + (-6) - 4 = -4
14. 3 + (-3) + 8 = 8
15. 5 + (-5) = 0
16. 4 + (-4) + (-6) = -6
17. 3 + (-3) + 7 = 7
18. 2 + (-2) + (-3) = -3
19. 1 + (-1) + 5 = 5
20. 4 + (-4) + (-2) = -2

Page 13 - 14
1. 3 + (-10) = -7
2. 10 + (-6) = 4
3. 9 + (-7) = 2
4. 4 + (-10) = -6
5. 10 + (-5) = 5
6. 5 + (-4) = 1
7. -8 + 0 = -8
8. -2 + 10 = 8
9. -7 + 10 = 3
10. -9 + 5 = -4
11. -7 + 2 = -5
12. -10 + 1 = -9
13. 1 - (-1) = 2
14. 8 - (-1) = 9
15. 4 - (-4) = 8
16. 3 - (-2) = 5
17. 7 - (-1) = 8
18. 2 - (-8) = 10
19. 0 - (-3) = 3
20. -6 - (-2) = -4
21. -9 - (-9) = 0
22. -5 - (-8) = 3
23. -10 - (-7) = -3
24. -7 - (-3) = 4

Page 15
1. -3 + (-5) = -8
2. -9 + (-2) = -11
3. 8 + (-2) = 6
4. -5 + (-8) = -13
5. -3 + 5 = 2
6. -9 + 4 = -5
7. -4 + 9 = 5
8. 10 + -8 = 2

Page 15 (cont)
9. 6 + (-13) = -7
10. 13 + -9 = 4
11. -5 + (-7) = -12
12. -13 + 5 = -8
13. 13 + (-4) = 9
14. -8 + (-9) = -17
15. -5 + (-6) = -11
16. -14 + 8 = -6
17. -1 + 10 = 9
18. 9 + (-3) = 6
19. 10 + (-20) = -10
20. -2 + 15 = 13
21. -12 + 30 = 18
22. -5 + -6 = -11
23. 0 + (-8) = -8
24. -12 + -3 = -15
25. -4 - 10 = -14
26. 5 - 9 = -4
27. -1 - 6 = -7
28. -8 - 8 = -16
29. -6 - (-5) = -1
30. 12 - (-3) = 15
31. 5 - (-8) = 13
32. -8 - (-3) = -5
33. -6 - (-7) = 1
34. 9 - (-5) = 14
35. -6 - (-4) = -2
36. -7 - (-7) = 0
37. -11 - 6 = -17
38. -15 - (-8) = -7
39. -3 - 15 = -18
40. 0 - (-9) = 9
41. -14 - (-4) = -10
42. -3 - (-9) = 6
43. -9 - (-6) = -3
44. 1 - (-10) = 11
45. 15 - (-5) = 20
46. (-5) - 12 -17
47. -7 - (-5) -12
48. 20 + (-5) = 15
49. 8 + (5) = 13
50. -9 + (-5) = -14

Page 16 - 22
1. a. 100 - 44 = 56 years old
   b. 69 - 30 = 39 years old
   c. 100 - 69 = 31 years
2. 138 m = 123 m + 15 m
   Rock has fallen 138m
3. Difference between -31 and 156
   seconds is 187sec (3m 7sec).
4. -50 - 14 = -64 (64°C difference)
5. 10207m - 6000m = 4207m
6. ($10 500 + $5 400 - $6 100
   - $12 800 + $4 200 + 2 500)
   = $3 700 overall profit

30

Page 16 - 22

7. Profit $10 000
8. **a.** $100 million
   b. -$20m + $70m = $50 ($70m profit)
   c. -$100 million ($100m loss)
9. + 2 + 1 - 5 + 3 = 1 metre rise
10. $180 + $750 - $550 = $380
11. -$467 + $129 = -$338 (owes $338)
12. -$470 + $45 - $160 + $500 = -$85
13. 27°C - (-128°C) = 155°C
14. 0 (par) + 0 - 2 - 2 + 4 + 2 = 2
    2 shots over = 31 shots
15. 2 + (-7) = -5
16. -5 + 2 = -3
17. -11 + 5 = -6
18. 4 + (-2) = 2
19. -6 + 7 = 1
20. 8 - (-8) = 16
21. 3 - (-5) = 8
22. -4 - (-6) = 2
23. -7 - 0 = -7
24. -13 - 2 = -15
25. 0 + (-27) = -27
26. -17 + (-25) = -42
27. -24 + (-9) = -33
28. 19 + (-15) = 4
29. -27 + 18 = -9
30. 53 - (-18) = 71
31. -15 - (-45) = 30
32. -23 - (-16) = -7
33. -33 - 44 = -77
34. -20 - (-20) = 0
35. -26 + 15 = -11
36. -31 + (-12) = -43
37. 13 + (-9) = 4
38. 27 + (-4) = 23
39. -13 - 16 + 36 - 11 + 19 + 20 = 35
    Up 35 points for the 6 weeks.
40. 0 (zero) as when added, each number's
    opposite cancel out each other.
41. 18 + (-21) = -3
42. 23 - 38 = -15
43. -21 + 25 = 4
44. -50 - (-9) = -41
45. 23 - (-23) = 46
46. -32 - 10 = -42
47. 25 + (-31) = -6
48. -19 - 8 = -27
49. 29 - (-16) = 45
50. -15 - 21 = -36

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1. Time taken = 28 minutes
   -2°C × 28 = -56°C
   15°C - 56°C = -41°C
2. **a.** -3°C
   b. 7 × (-3°C) = -21°C
   c. 10 × (-3°C) = -30°C
   Starting temperature = 20°C
   Temperature decreases by -30°C
   Final temperature 20 - 30 = -10°C
3. 1400°C ÷ 280°C/second = 5 seconds
4. **a.** 2100 ÷ 300 = 7, boiling point decreases
   by 7°C, so water boils at 93°C
   b. 3600 ÷ 300 = 12, boiling point
   decreases by 12°C, so water boils at 88°C
5. -(19 + 21 + 19 + 22 + 24 + 21) ÷ 6
   -116 ÷ 6 = -21°C
6. 8 × -11 = -88
7. -6 × -8 = 48
8. -5 × -7 = 35
9. 0 × -20 = 0
10. 20 × -6 = -120
11. -9 × -4 = 36
12. -4 × 7 = -28
13. -15 × 3 = -45
14. 4 × -18 = -72
15. 25 × -5 = -125
16. -24 ÷ 8 = -3
17. 0 ÷ -20 = 0
18. -36 ÷ -6 = 6
19. 28 ÷ -7 = -4
20. 40 ÷ -4 = -10
21. -39 ÷ -13 = 3
22. 96 ÷ -3 = -32
23. -42 ÷ -7 = 6
24. -98 ÷ 2 = -49
25. -64 ÷ 16 = -4
26. -15 × -4 = 60
27. 24 ÷ -6 = -4
28. -18 × 3 = -54
29. -54 ÷ -2 = 27
30. 20 × -7 = -140
31. -200 ÷ -5 = 40
32. -90 × -9 = 810
33. -76 ÷ 4 = -19
34. -6 × -5 ÷ -3 = -10
35. -8 ÷ 2 × -3 = 12
36. Week   1   2   3   4
    H - A   2°C   3°C   -1°C   4°C
    Mean of differences
    (2 + 3 + (-1) + 4) ÷ 4 = 8 ÷ 4
                          = 2°C
37. 25 m/min × 240 min = 6000 metres
    -425 m ÷ 25 m/min = 17 minutes

38. (-3 × 5) × 4 = -60
39. (8 × -6) × -2 = 96
40. (-6 × -5) × -2 = -60
41. -2 × (-8 × -2) = -32
42. (-7 × -2) × -3 = -42
43. -3 × (9 × 10) = -270
44. (4 - 3) × (5 - -1) = 6
45. (-5 - 8) × (6 - 10) = 52
46. -10 × (-5 × -5) = -250
47. 75 ÷ (-5 × -5) = 3
48. -(8 × -5 × -5) = -200
49. (4 × -5 × -5) ÷ -10 = -10
50. (-5 × -5) + (5 × -5 × -5) = 150

Types of Integers

Negative	Positive
Temperature below zero	Temperature above zero
Below sea level	Above sea level
Deposit	Withdrawal
Debt (owing) money	Having money in the bank
Loss	Profit
Basement car parks	Floors above ground level
Negative(-) atoms	Positive (+) atoms
Loss	Gain
Decrease	Increase
Down	Up
Backward	Forward
Dive	Rise
Descending	Ascending
Falling	Clinbing
BC	AD